Mock Paper Mark Scheme

Advanced Subsidiary/Advanced GCE
General Certificate of Education

Question number	Scheme	Marks
1.	H_0: $\mu = 100$, H_1: $\mu > 100$	B1
	$\bar{x} = \frac{710.9}{7} = 101.5571\ldots$; $s^2 = \frac{72219.45 - \frac{(710.9)^2}{7}}{6}$	B1; M1
	$s^2 = 3.746\ldots$ or $s = 1.9355$	A1
	test statistic $t = \frac{101.557 - 100}{\frac{1.936}{\sqrt{7}}}$ = awrt 2.13	M1 A1
	t_6 5% 1-tail critical value = 1.943	B1 √
	Significant result. Reject H_0, there is evidence that the mean weight is more than 100g.	A1 (8)
		(8 marks)
2.	D = dry—wet H_0: $\mu_D = 0$, H_1: $\mu_D \neq 0$	B1
	d : 0.6, –1, –1.9, –1.4, –1.3, 0.5, –1.6, –0.6, –1.8	M1
	$\bar{d}$: $-\frac{8.5}{9} = -0.9\dot{4}$, $s_d^2 = \frac{15.03 - 9 \times (\bar{d})^2}{8} = 0.87527\ldots$	A1, A1
	$t = \frac{-0.9\dot{4}}{\frac{s_d}{\sqrt{9}}}$ = awrt –3.03	M1 A1
	t_8 2-tail 1% critical value = 3.355	B1
	Not significant – insufficient evidence of a difference between mean strength	A1 √ (8)
		(8 marks)

Question number	Scheme	Marks
3. (*a*)	H_0: $p = 0.3$ (or 0.7) H_1: $p < 0.3$ (or > 0.7)	B1 (1)
(*b*)	Let X = number who contract virus. Under H_0 $X \sim B(30, 0.3)$	
	P(Type I error) = $P(X < 6 \mid p = 0.30)$ = $P(X \le 5)$ = 0.0766	M1 A1 (2)
(*c*)	(i) Power = $P(Y \le 5 \mid Y \sim B(30, 0.2))$ = 0.4275	M1 A1
	(ii) Power = $P(Y \le 5 \mid Y \sim B(30, 0.1))$ = 0.9268	A1 (3)
(*d*)	Let C = number who contract virus. Under H_0 $C \sim B(50, 0.3)$	
	We require c such that $P(C \le c) \approx 0.05$	M1
	$P(C \le 10) = 0.0789$, $P(C \le 9) = 0.0402$ $\therefore$ critical region is $C \le 9$	A1 (2)
(*e*)	Size = 0.0402	B1 (1)
(*f*)	(i) Power = $P(D \le 9 \mid D \sim B(50, 0.2))$ = 0.4437	B1
	(ii) Power = $P(D \le 9 \mid D \sim B(50, 0.1))$ = 0.9755	B1 (2)
(*g*)	Advantage: second test is more powerful	B1
	Disadvantage: second test involves greater sample size, $\therefore$ more expensive or takes longer	B1 (2)
		(13 marks)

Question number	Scheme	Marks
4. (*a*)	$\bar{x} = \dfrac{847.89}{10} = 84.79$; $s_x^2 = \dfrac{103712.6151 - (847.89)^2/10}{9}$	B1
	$s_x^2 = 3535.6522...$	B1
	or $s_x = 59.461\ldots$	
	2.262	B1
	95% confidence interval for $\mu = 84.79 \pm 2.262 \times \dfrac{59.461}{\sqrt{10}} = (42.25, 127.33)$	
	accept (42.3, 127.3)	M1, A1, A1 (6)
(*b*)	95% confidence interval $\bar{x} \pm 1.96\dfrac{\sigma}{\sqrt{n}}$	
	so chief accountant requires $1.96\dfrac{\sigma}{\sqrt{n}} < 10$	M1 A1
	i.e. $\dfrac{\sigma^2}{n} < \left(\dfrac{10}{1.96}\right)^2 = 26.0308\ldots = 26.03$ (2 d.p.)	A1 cso (3)
(*c*)	Require the upper confidence limit of 98% confidence interval for σ^2	
	$\chi_9^2 = 2.088$; i.e. $\dfrac{9s^2}{\sigma^2} > 2.088$, $\Rightarrow \sigma^2 < 15239.88\ldots$ awrt 15240	B1; M1, A1 (3)
(*d*)	Substitute into part (*b*), $n > \dfrac{15240}{26.03} \Rightarrow n = 586$	M1, A1 (2)
		(14 marks)

Question number	Scheme	Marks
5. (*a*)	(i) $H_0: \sigma_C^2 = \sigma_N^2$, $H_1: \sigma_C^2 > \sigma_N^2$	B1
	$\frac{s_C^2}{s_N^2} = \frac{5.7^2}{3.5^2} = 2.652\ldots$; $F_{8,9}$ (5%) critical value = 3.23	M1; B1
	Not significant so do not reject H_0 – insufficient evidence that variance using conventional method is greater	A1 √ (4)
	(ii) $H_0: \mu_N = \mu_C$, $H_1: \mu_N > \mu_C$	B1
	$s^2 = \frac{8\times5.7^2 + 9\times3.5^2}{17} = \frac{370.17}{17} = 21.774\ldots$	M1
	Test statistic $t = \frac{82.3 - 78.2}{\sqrt{21.774...(\frac{1}{9}+\frac{1}{10})}} = 1.9122\ldots$ awrt 1.91	M1 A1
	t_{17} (5%) 1-tail critical value = 1.740	B1
	Significant – reject H_0. There is evidence that new style leads to an increase in mean	A1 √ (6)
(*b*)	Assumed population of marks obtained were normally distributed	B1 (1)
(*c*)	Unbiased estimate of common variance is s^2 in (ii)	
	$7.564 < \frac{17s^2}{\sigma^2} < 30.191$	B1 M1 B1
	$\sigma^2 > \frac{17\times21.774...}{30.191} = 12.3$ (1 d.p.)	A1
	$\sigma^2 < \frac{17\times21.774...}{7.564} = 48.9$ (1 d.p.)	A1 (5)
		(16 marks)

Question number	Scheme	Marks	
6. (*a*)	$X_1 \sim B(10, p)\ \therefore\ E(X_1) = 10p \Rightarrow E(R_1) = E\left(\frac{X_1}{10}\right) = \frac{10p}{10} = p$	B1	(1)
(*b*)	$X_2 \sim B(n, p)\ \therefore\ E(X_2) = np \Rightarrow E(R_2) = E\left(\frac{X_2}{n}\right) = \frac{np}{n} = p$	B1	
	$E(Y) = E(\frac{1}{2}[R_1 + R_2]) = \frac{1}{2}[E(R_1) + E(R_2)] = \frac{1}{2}[p + p] = p$	B1	(2)
(*c*)	$\text{Var}(R_2) = \frac{1}{n^2}\text{Var}(X_2) = \frac{np(1-p)}{n^2} = \frac{p(1-p)}{n}$	B1	
	$\text{Var}(R_1) = \frac{p(1-p)}{10}\quad \therefore\ \text{Var}(Y) = \frac{1}{4}[\text{Var}(R_1) + \text{Var}(R_2)],$	M1	
	$= \frac{1}{4}\left[\frac{p(1-p)}{10} + \frac{p(1-p)}{n}\right]$	A1	(3)
(*d*)	Since $\text{Var}(R_2) = \frac{p(1-p)}{n} \to 0$ as $n \to \infty$, $\therefore\ R_2$ is consistent	M1, A1	(2)
(*e*)	$\text{Var}(R_1) = \frac{p(1-p)}{10} > \frac{p(1-p)}{20} = \text{Var}(R_2)$		
	$\text{Var}(Y) = \frac{p(1-p)}{4}\left[\frac{1}{10} + \frac{1}{20}\right] = \frac{p(1-p)}{80} \times 3 < \text{Var}(R_2)$	M1	
	Since all 3 are unbiased, we select the one with minimum variance, i.e. Y	A1	(2)

Question number	Scheme	Marks
(*f*)	$X_1 + X_2 \sim \text{B}(n + 10, p)$ so consider $\dfrac{X_1 + X_2}{n+10}$	B1
	$\text{E}\left(\dfrac{X_1 + X_2}{n+10}\right) = \dfrac{(n+10)p}{(n+10)} = p$ (show unbiased)	M1
	$\text{Var}\left(\dfrac{X_1 + X_2}{n+10}\right) = \dfrac{p(1-p)}{n+10}$ (find variance)	M1
	$\dfrac{p(1-p)}{n+10} < \dfrac{p(1-p)}{10}$ $\therefore$ always better than R_1 and both $\dfrac{p(1-p)}{n+10} < \dfrac{p(1-p)}{n}$ $\therefore$ always better than R_2	A1
	$\dfrac{p(1-p)}{n+10} < \dfrac{p(1-p)}{4}\left[\dfrac{n+10}{10n}\right]$ $\Leftrightarrow$ $40n < 100 + 20n + n^2$ $\Leftrightarrow$ $0 < 10^2 - 20n + n^2$ $\Leftrightarrow$ $0 < (10 - n)^2$ Show better than Y Use of $n = 20$ acceptable	M1
	$\therefore$ $\dfrac{X_1 + X_2}{n+10}$ is unbiased and always has smaller variance	A1 cso (6) **(16 marks)**